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Glassy phases in interacting disordered systems

ABSTRACT

In the thesis, a theoretical description of the system of strongly interacting bosons in the presence of off-diagonal disorder was formulated, along with obtaining its phase diagram and characterizing the phases. The considered system combines two types of ordering – superfluidity and glassiness – which allows one to study the phases containing each of these orders separately, as well as the one in which both coexist.

The first two chapters of the dissertation are an introduction to the problem. The first one describes the considered system. First, two pillars are introduced – spin glasses and strongly interacting bosons. Then, possible ways of combining their features in one system are discussed. In the second chapter, the most important elements of the theoretical description of the system are discussed: the Landau theory of phase transitions, order parameters for glassiness and superfluidity, as well as formalism and analytical methods used.

In the third chapter, an analytical derivation is presented, in which the free energy of the system is transformed using, among others, the replica method and the Trotter-Suzuki method. The effective form of the free energy is obtained, and, from the condition of its minimization – a system of self-consistent equations for the values of order parameters and other quantities characterizing the system. The derivation is carried out twice – first, in a simplified model assuming zero mean of the hopping integral and vanishing order parameters (which is sufficient to describe the disordered phase). Then - in the full model without these simplifications. The first approach allows focusing on the most important effects of disorder (and more efficient numerical calculations due to the simplicity of the model), while the second approach allows for the analysis of superfluid phases (including the identification of a new superglass phase) and examining the influence of all available parameters on the phase diagram.

The fourth chapter describes the most important elements of adapting the obtained self-consistent equations for their numerical solution. Attention is mainly devoted to the issue of fast calculation of thermodynamic averages. A proof is shown that it is impossible to compute these averages using the Monte Carlo method, making it necessary to compute a sum with the number of terms increasing exponentially with the accuracy of the solution. Nevertheless, the symmetries of the problem make it possible to speed up this summation.

The fifth chapter is devoted to the problem of the stability of the symmetric solution in the replica space. The replica method is one of the tools used in the analytical conversion of free energy. At a certain stage of the derivation, it was necessary to assume the symmetry of the solution in the replica space. It is known from the spin glass literature that such a simplification gives the correct solution in the phase without the glassy order but corresponds to a maximum instead of a minimum of the free energy when this order is present. Although it is difficult to obtain a symmetry-free solution, checking the correctness of a symmetric solution is viable. Such a test allows for determining the boundary between the glassy order and the lack of it, including where it is not possible based on Landau's theory.

The last two chapters contain the numerical results and their analysis. The sixth chapter contains phase diagrams from both the simplified and the full model. Various two-dimensional cross-sections are shown, including those showing the dependence on the chemical potential, allowing one to refer to the literature on bosonic systems without disorder and with diagonal disorder. Moreover, diagrams showing the temperature dependence are presented, featuring a quantum phase transition and allowing for a comparison to literature on spin glasses. The seventh chapter is devoted to the quantities that characterize the obtained phases. It consists of an analysis of: values of order parameters, including the observation of anti-correlation of glass and superfluid order parameters in the superglass phase; dynamic correlation functions to determine the presence or absence of system memory; compressibility, which is a typical quantity used to characterize the phase transition between a Mott insulator and a superfluid phase; as well as the susceptibility of the order parameter and its critical exponent.

The dissertation concludes with a summary, which lists the most important achievements described in the dissertation and outlines possible further research directions.